Corrigé TD 6

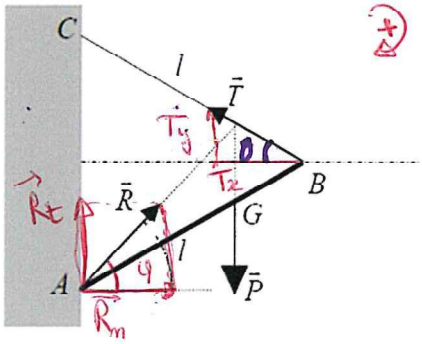
Exercice 5

$AB = l$

$AG = a$ (tige non homogène)

coefficient de frottement

$f = \tan\varphi$



* Exprimer $\tan\theta$ en fonction de a, l et f en cas d'équilibre

Bilan de force : $\vec{R}$, $\vec{P}$ et $\vec{T}$

$\vec{R} = \vec{R_n} + \vec{R_t}$

et $R_t = f R_n = \tan\varphi R_n$

En cas d'équilibre : $\sum \vec{F} = \vec{0}$

$\sum \vec{M}_A(\vec{F}) = \vec{0}$

$\sum \vec{F} = \vec{0}$

$\Rightarrow \vec{R} + \vec{P} + \vec{T} = 0$

$\cos\theta = \frac{T_x}{T} \Rightarrow T_x = T\cos\theta$

$\sin\theta = \frac{T_y}{T} \Rightarrow T_y = T\sin\theta$

projection selon la verticale :

$R_t - P + T\sin\theta = 0$

$\sin\varphi = \frac{R_t}{R} \Rightarrow R_t = R\sin\varphi$

$\boxed{R\sin\varphi - P + T\sin\theta = 0}$ ①

projection selon l'horizontale :

$R_n + 0 - T\cos\theta = 0$

$\cos\varphi = \frac{R_n}{R} \Rightarrow R_n = R\cos\varphi$

$\boxed{R\cos\varphi - T\cos\theta = 0}$ ②

② $\Rightarrow R\cos\varphi = T\cos\theta \Rightarrow \boxed{R = \frac{T\cos\theta}{\cos\varphi}}$ ③

je remplace ③ dans ① :

$$\frac{T\cos\theta\,\sin\varphi}{\cos\varphi} - P + T\sin\theta = 0$$

$$T\cos\theta\,\tan\varphi - P + T\sin\theta = 0$$

$\tan\varphi = f$

$$\boxed{P = f.T\cos\theta + T\sin\theta} \quad ④$$

$$\sum \vec{\mathcal{M}}_A(\vec{F}) = \vec{0}$$

$$\vec{\mathcal{M}}_A(\vec{R}) + \vec{\mathcal{M}}_A(\vec{P}) + \vec{\mathcal{M}}_A(\vec{T}) = \vec{0}$$

($\vec{R}$ passe par A) $\Rightarrow \vec{\mathcal{M}}_A(\vec{R}) = \vec{0}$

$$0 + P.d - T.d_2 = 0$$

$$\Rightarrow P.a.\cos\theta - T.\ell.\sin 2\theta = 0$$

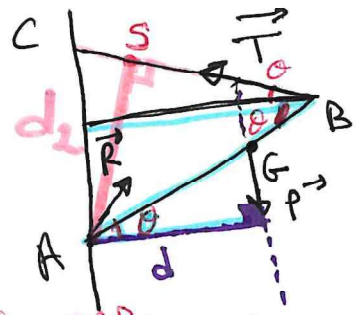
$\sin 2\theta = 2\sin\theta\cos\theta$

$$P.a.\cos\theta - T.\ell.2.\sin\theta.\cos\theta = 0$$

$$P.a - T.\ell.2.\sin\theta = 0$$

$$P.a = 2\ell.T.\sin\theta$$

$$\boxed{P = 2\frac{\ell}{a}.T.\sin\theta} \quad ⑤$$



ACB isocèle car CB = BA =

$G\hat{A}P = \theta$ (égalité angle alterne interne)

Dans le Δ GAP :

$\cos\theta = \frac{d}{AG} \Rightarrow d = AG\cos\theta$

$\boxed{d = a\cos\theta}$

Dans le Δ ASB :

l'angle $S\hat{B}A = 2\theta$

$\sin 2\theta = \frac{d_2}{AB} \Rightarrow d_2 = AB\sin 2\theta$

$\boxed{d_2 = \ell\sin 2\theta}$

Relation trigo :

$\sin 2\theta = 2\sin\theta\cos\theta$

④ = ⑤ $\quad P = P$

$$\Rightarrow f.\cancel{T}.\cos\theta + \cancel{T}\sin\theta = 2\frac{\ell}{a}.\cancel{T}.\sin\theta$$

$:\cos\theta$ ↓

$$f + \tan\theta = 2.\frac{\ell}{a}\tan\theta$$

$$\Rightarrow \tan\theta - \frac{2\ell}{a}\tan\theta = -f \Rightarrow \tan\theta\left(1 - \frac{2\ell}{a}\right) = -f \Rightarrow \boxed{\tan\theta = \frac{-f}{1 - \frac{2\ell}{a}} = \frac{f}{\frac{2\ell}{a} - 1}}$$